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The impact of network topology on delay bound in wireless Ad Hoc networks

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Abstract We consider single channel wireless networks with interference constraint among the links that can be activated simultaneously. The traffic flows are assumed to be single hop. Delay performance of the well known throughput optimal maximum weight link scheduling algorithm has been studied recently. In this paper, we study the relation between network topology and delay of maximum weight link scheduling algorithm. First, we consider 1-hop interference model. Under this interference model, an upper bound for the average delay of packets is derived analytically in terms of edge chromatic number of the network graph. Then the results have been extended to the case of general interference model. Under this model of interference, an upper bound for delay as a function of chromatic number of conflict graph is derived. Since chromatic number and edge chromatic number are network topology parameters, the results show that how the upper bound of delay is affected by network topology. Simulation results confirm our analytical relations.

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1 Introduction

In single channel wireless networks, links need to be scheduled for data transmission due to the presence of interference among them. Link scheduling has been known to be a challenging problem when the design objective is to optimize achievable throughput (capacity). Let us assume that associated to each link is a queue and packets are queued before they are transmitted over the link. A throughput optimal maximum weight (also known as Max-Weight) scheduling algorithm has been devised in [1]. The optimization problem is to choose a set of non-interfering links with the maximum sum of weights, where the link weights are the queue sizes on the links. The proposed solution however, needs that a central and high computational complexity algorithm to be executed at each time slot in the network. The complexity of the algorithm is highly affected by the *interference model* used to deal with interference and is a NP-Hard problem in general case [2].

Many research efforts then conducted in this area to develop scheduling algorithms with lower complexity which are more applicable in wireless networks without central coordinator node, like Ad Hoc networks. A class of randomized algorithms with linear complexity has been proposed by Tassiulas in [3]. These type of central algorithms achieve the entire throughput region under some mild assumption. A distributed version of [3] has been devised by the use of *gossiping* technique with the assumption that interference model is *node exclusive* [4].

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Another distributed implementation of [3] under node exclusive interference model (also known as 1-hop interference model) and for the case of single hop traffic flows has been developed by Bui et al. which is shown to attain a fraction $\frac{k}{k+2}$ of throughput region, where k is a tunable parameter of the algorithm [5]. This work was extended to include multi hop traffic as well in [6]. Subsequently, a similar approach was used to develop a link scheduling algorithm with $O(1)$ complexity for M-hop interference model [7].

Another approach to implement distributed link scheduling algorithm is to apply *random access* and *backoff time* technique. Q-SHED is an example of a link scheduling algorithm that uses this approach [8].

Link scheduling algorithm under 1-hop interference model is equivalent to maximum weight matching in graph theory which suffers high computational complexity. Then, a trend of research study in graph theory is to devise approximation algorithms with low complexity that achieves a fraction of optimal solution. A simple approximation algorithm for maximum weight matching problem is *greedy* algorithm that guarantees to achieve a fraction $\frac{1}{2}$ of optimal solution (maximum weight matching) [9]. The greedy algorithm begins with an empty set and extends it in each round by adding the heaviest edge currently available. A distributed version of greedy algorithm has been proposed in [10]. Another algorithm incurring less complexity and the same fraction of optimal solution has been proposed in [11]. In [12] a linear time approximation algorithm for maximum weight matching problem has been devised that attains a fraction of $\frac{2}{3} - \epsilon$ of optimal solution.

Maximal scheduling is another trivial approximation to Max-Weight scheduling. Maximal scheduling means that if the queue of a link l is non-empty, then either l is selected for transmission, or some other links within the interference set of l is selected, where the interference set of l is a set of links that interferes with l in addition to l itself. Maximal scheduling is of interest due to its localized nature, low overhead and complexity, and then ease of distributed implementation. Maximal scheduling is known to achieve throughput that is within a constant factor of optimality for general interference model [13, 14].

While most of the previous works focused on the throughput performance of scheduling algorithms, recently the delay property of these algorithms has emerged as a new design parameter when delay sensitive applications are growing rapidly. These applications require that the network provides some kind of guarantee for the upper bound of delay. Link scheduling algorithms (in MAC Layer) in addition to routing protocols (in Network Layer) affect the delay of packets in the network [15]. However, for the case of single hop traffic where packets traverse only one hop and then leave the network, the delay behavior of the network is mostly determined by link scheduling algorithms.

Neely has demonstrated that the delay of Max-Weight link scheduling algorithm in cellular networks with ON/OFF links is *order optimal* (i.e. independent of network size) [16, 17]. Also, the order optimal delay property of Max-Weight scheduling in wireless Ad Hoc networks with single hop traffic flows has been derived in [18] provided that network topology is sparse and uniform. It has been shown that the average delay of the maximal scheduling algorithm in wireless networks with single hop traffic is independent of network size, and hence is order optimal [19]. A lower bound and upper bound of delay in wireless networks with single hop traffic and general interference constraint has been derived in [20]. The derived upper bound is associated to a generalized version of Max-Weight schedule termed as GMWM which is a kind of weighted MWS. The work was extended to consider multi hop traffic in [21].

In this paper, we aim to generalize some of previous works by considering the impact of network topology on the delay.

In a recent paper, a frame-based scheduling algorithm for mmWave (millimeter wave) WPAN (Wirelee Personal Area Network) has been proposed which is based on greedy graph coloring technique [22]. The main objective of the paper is to leverage collision-free concurrent transmissions to fully exploit spatial reuse in such networks which obtained by using directional antennas. This property allows the designers to consider 1-hop interference model for WPANs such as Bluetooth network. In WPANs, there is a central node, termed as piconet coordinator, which is responsible for collecting traffic demands from other nodes and computes schedules to enable concurrent directional transmissions in the WPAN. Due to the property of directional transmissions, the topology of these kind of networks is not fixed and can be controlled by the piconet to fulfill different performance goals such as minimizing delay and maximizing network throughput [22]. The relation between the mentioned paper and our work is as follows. The Piconet can determine the topology of the WPAN network, while we derive an upper bound of delay for similar networks which is related to the topology of the network. Therefore, our derived upper bound of delay can be used by the piconet to evaluate the delay performance of the constructed network, provided that topology variation occurs infrequently.

The main contribution of this paper is to analytically derive two upper bounds for the network average delay which is explicitly related to the network topology. One of the derived upper bounds is associated to networks with 1-hop interference model while the other one is associated to networks with general M-hop interference model. Specifically, we show the relation between edge-chromatic number of the network graph and the average delay that packets incur in the network, while the interference constraint is applied by 1-hop interference model. As edge-chromatic number depends on the topology of the network, we indeed derive the relation between topology and delay in the order sense. The result is extended to M-hop interference model by using the chromatic number of network's conflict graph.

Simulation results support our analytical findings. To the best of our knowledge, this is the first time that the concepts of graph coloring in relation with average delay in wireless networks are studied. The results of this paper show the impact of network topology on upper bounds of delay. These bounds can be used to provide some level of assurance for delay sensitive applications in the network.

The rest of the paper is organized as follows. Sect. 2 introduces the system model and the formal problem definition. In Sect. 3 the upper bound of the delay for node exclusive interference model is derived analytically. In Sect. 4 the simulation results are used to validate analytical relations. The extension of the work to general interference model is developed in Sect. 5 In Sect. 6 we provide a discussion on the importance of the results as well as a comparison to a recently published article. Finally, some concluding remarks are provided in Sect. 7.

2 System model

A wireless Ad Hoc network can be modeled as a directed graph $G(\mathcal{N},\mathcal{L})$, where $\mathcal{N}$ and $\mathcal{L}$ denote the sets of nodes and links respectively; N and L are used to denote the cardinalities of $\mathcal{N}$ and $\mathcal{L}$. We assume that the network is stationary and nodes are globally synchronized. Time is slotted and all packets have the same normalized size such that one packet can be transmitted within one time slot. A time slot consists of a scheduling duration, followed by a transmission duration. In the same spirit of existing works [2, 4, 5], we assume that the wireless network is single-channel and single-radio.

In this paper we adopt both 1-hop and general interference models [2]. The 1-hop interference model is also known as node exclusive model in the literature. Link scheduling under 1-hop interference model results in finding a *matching* in the network graph $G(\mathcal{N},\mathcal{L})$. A matching in a graph is a set of edges with no shared end nodes. This is a commonly used model for Bluetooth and FH-CDMA systems [7]. To demonstrate general interference constraints among links of the network, we use another well known graph, which is called *conflict graph*.

Definition 1 (**Conflict Graph**) Let $G'(\mathcal{L},\mathcal{E})$ denotes the conflict graph, where $\mathcal{L}$ and $\mathcal{E}$ are sets of nodes and edges respectively. Corresponding to each edge in $G(\mathcal{N},\mathcal{L})$, there is a node in $G'(\mathcal{L},\mathcal{E})$. Then, the set of nodes in G' is the same as the set of edges (links) in G. Two nodes in G' are adjacent whenever their corresponding edges in G are interfering with each other. M-hop interference model [2] is a special case of general interference model where links within M-hop distance of each other are considered as interfering links.

As we focus on link scheduling, we consider only single hop flows. The results however can be extended to multi-hop traffic, using the back-pressure approach [1] as the routing component of the joint scheduling and routing problem.

Associated with each link l, we assume that the stochastic process $A_l(t)$ denotes the number of packets arrived at time slot t. We assume that the second moment of $A_l(t)$ is finite. Let $\boldsymbol{A}(t)$ be the vector of arrivals at time slot t among all the links. Throughout this paper, we use bold symbols $\boldsymbol{X}$ to describe a column vector with elements X_l.

We treat $A_l(.)$ as an *i.i.d* process over time and denote $\lambda_l = \mathbb{E}[A_l(t)]$. Let $Q_l(t)$ be the queue length of link l. The binary vector $\boldsymbol{I}(t)$ of length L is used to denote the set of *active links* at time slot t, where $I_l(t) = 1$ if link l is scheduled at time slot t and has a positive queue length. Then, the queue length dynamics can be represented as follows.

$$\boldsymbol{Q}(t+1) = \boldsymbol{Q}(t) + \boldsymbol{A}(t) - \boldsymbol{I}(t). \tag{1}$$

We adopt the definition of *Stability* in [23]. A system is strongly stable if

$$\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\left[\sum_{l\in\mathcal{L}} Q_l(\tau)\right] < \infty. \tag{2}$$

Let $\mathcal{S}$ be the set of all feasible schedule vectors $\boldsymbol{I}$ in G, i.e., $\mathcal{S} = \{\boldsymbol{I}^1, \boldsymbol{I}^2, \ldots, \boldsymbol{I}^{|\mathcal{S}|}\}$. The *throughput-region* under a given scheduling algorithm is a set of all offered load vector $\boldsymbol{\lambda}$ under which the system remains stable. The throughput-region $\Lambda \subset \mathcal{R}_+^L$ of the system is defined as the union of the throughput-regions under all scheduling algorithms in $\mathcal{S}$. In other words, Λ denotes the set of $\boldsymbol{\lambda}$s for which there exists a scheduling that can stabilize the system. It is known that Λ coincides with the convex hull (convex envelope) of all possible schedules and the following result was shown in [1].

If $\boldsymbol{\lambda}$ belongs to the interior of Λ, then the network is stable under a scheduling algorithm that selects links for transmission according to the Max-Weight schedule in Eq. (3).

$$\boldsymbol{I}^*(t) = \arg\max_{\boldsymbol{I}\in\mathcal{S}} \boldsymbol{I}.\boldsymbol{Q}(t). \tag{3}$$

where $\boldsymbol{I}^*(t)$ is denoted by *optimal schedule* at time slot t. As a matter of notational simplicity, we omit the transpose symbol in inner product of two vectors, $\boldsymbol{I}$ and $\boldsymbol{Q}(t)$ above, and the rest of this paper. According to Eq. (3), Max-Weight algorithm indicates that at each time slot t, a set of non-interfering links should be chosen that has the maximum sum of queue sizes (maximum sum of weights) on the links.

In the following sections, we derive two upper bounds for the average delay of the system which reflect the impact of network topology, provided that arrival traffic is strictly inside the throughput region and optimal scheduling algorithm is deployed. To this end, we use two topological parameter, *edge-chromatic number* of the network graph denoted by $\chi'(G)$ and *chromatic number* of conflict graph denoted by $\chi(G')$. The definitions of these terms are as follows.

Definition 2 (**k-edge-colorable**) [24] A proper *k-edge-coloring* of G is a labeling (coloring) $f : \mathcal{L} \rightarrow \omega$, where $|\omega| = k$ and incident edges have different labels (colors). The edges of one color form a *matching* in the graph. The graph is *k-edge-colorable* if it has a proper k-edge-coloring.

Definition 3 (**edge-chromatic number**) [24] The *edge-chromatic number* $\chi'(G)$ of a graph G is the least k such that G is k-edge-colorable.

Definition 4 (**k-colorable**) [24] A proper *k-coloring* of G is a labeling (coloring) $f : \mathcal{N} \rightarrow C$, where $|C| = k$ and adjacent nodes have different labels (colors). The nodes of one color form an *independent set* in the graph. An independent set in a graph is a set of nodes that no two of which are adjacent. The graph is *k-colorable* if it has a proper k-coloring.

Definition 5 (**chromatic number**) [24] The *chromatic number* $\chi(G)$ of a graph G is the least k such that G is k-colorable.

We mean by the word "delay" throughout the paper, the average delay of the packets in the network.

3 Delay analysis

In this section, we use the Lyapunov drift technique to obtain an upper delay bound for Max-Weight scheduling algorithm. To this end, we need to introduce the following theorem which will be the cornerstone for the rest of analysis.

Theorem 1 [23] *Let $\boldsymbol{Q}(t)$ be a vector process of queue backlogs that evolves according to some probability law, and let $V(\boldsymbol{Q}(t))$ be a non-negative function of $\boldsymbol{Q}(t)$. If there exists processes $f(t)$ and $g(t)$ such that the following inequality is satisfied for all time slots t,*

$$\begin{aligned} &\mathbb{E}[V(\boldsymbol{Q}(t+1)) - V(\boldsymbol{Q}(t))|\boldsymbol{Q}(t))] \\ &\quad \leq \mathbb{E}[g(t)|\boldsymbol{Q}(t)] - \mathbb{E}[f(t)|\boldsymbol{Q}(t)], \end{aligned} \tag{4}$$

then

$$\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{f(\tau)\} \leq \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\{g(\tau)\}.$$

Note that the relation between throughput region, Λ, and the set of all possible schedules, $\mathcal{S}$, is that Λ is the convex hull (convex envelope) of all feasible schedules I^k [1]. The convexity of Λ implies that for any vector $\theta \in \Lambda$, it can be written as $\theta = \sum_{I^k \in \mathcal{S}} \phi_k I^k$, where $\sum \phi_k \leq 1$. It has been shown that the necessary and sufficient condition for the stability of Max-Weight scheduling algorithm is that the arrival rate vector, λ, belongs to the interior (or strictly inside) the throughput region [1]. The interior of throughput region can be shown by $\rho\Lambda$, where $0 < \rho < 1$.

Therefore, if the arrival rate vector $\boldsymbol{\lambda}$ belongs to the interior of the throughput region, then there exists a loading factor $\rho < 1$ such that $\boldsymbol{\lambda} \in \rho\Lambda$. On the other hand, since throughput region is a convex hull, we can write

$$\boldsymbol{\lambda} = \rho \sum_{\boldsymbol{I}^k \in \mathcal{S}} \phi_k \boldsymbol{I}^k, \quad \text{and} \quad \sum_{k=1}^{|\mathcal{S}|} \phi_k \leq 1, \tag{5}$$

where $\phi_1, \phi_2, \ldots, \phi_{|\mathcal{S}|}$ are non-negative real numbers.

The parameter ρ shows how far the input rate vector is away from the throughput region boundary. We will see in Theorem 2 and Theorem 3 that as ρ approaches the value 1, which means that the arrival rate vector approaches the boundary of throughput region, the delay bound will be increased.

In this section, we adopt 1-hop interference model. Let us assume that the edges of the graph has been labeled by $\chi'(G)$ colors. Each color corresponds to a feasible schedule in the graph. Therefore, we have a set of feasible schedules Γ which consists of $\chi'(G)$ schedules. The union of active links in Γ equals $\mathcal{L}$ and any link belongs to only one schedule in the set. Note that $\Gamma \subset \mathcal{S}$, where $\mathcal{S}$ is the set of all feasible schedules. Let $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_{\chi'(G)}\}$, where $\boldsymbol{\gamma_i}$ is a schedule corresponds to color i, $1 \leq i \leq \chi'(G)$. Thanks to the convexity of Λ,

$$\frac{1}{\chi'(G)} \sum_{i=1}^{\chi'(G)} \gamma_i \in \Lambda \tag{6}$$

Let explain the intuition behind the above equation. Assume that a scheduling policy is defined such that it serves $\boldsymbol{\gamma_i}$s in a round robin fashion. Since the total number of $\boldsymbol{\gamma_i}$s is $\chi'(G)$, the average time between two consecutive service of a link is $\chi'(G)$. Then if all the arrival rates are

$\frac{1}{\chi'(G)}$, there exists a schedule to keep the system stable which means that the vector of arrivals belongs to the throughput region. Considering relation (6), convexity of Λ and the fact that $\sum_{I^k \in S} \phi_k I^k \in \Lambda$, we have

$$\rho \sum_{I^k \in S} \phi_k \boldsymbol{I}^k + \frac{1-\rho}{\chi'(G)} \sum_{i=1}^{\chi'(G)} \gamma_i \in \Lambda \tag{7}$$

Note that $\sum_{i=1}^{\chi'(G)} \boldsymbol{\gamma_i} = \mathbf{1}_L$, where $\mathbf{1}_L$ is a vector with all elements equal to 1. Then by using (5), relation (7) can be rewritten as follows,

$$\boldsymbol{\lambda} + \frac{1-\rho}{\chi'(G)} \mathbf{1}_L \in \Lambda \tag{8}$$

The derived relation indicates that the amount of value which can be added to the arrival rate vector while keeping the resulting vector inside the throughput region depends on two parameters : 1) the topology of the network which is denoted by edge chromatic number in the relation (8) and 2) the distance the arrival rate vector, $\boldsymbol{\lambda}$, is away from the boundary of region which is shown by the parameter ρ.

The following lemma will be used for the next step of analysis.

Lemma 1 *For any rate vector* $\boldsymbol{r}$ *strictly inside the throughput region, the following relation holds:*

$$\boldsymbol{Q}(t).\boldsymbol{r} < \boldsymbol{Q}(t).\boldsymbol{I}^*(t) \tag{9}$$

Proof Since $\boldsymbol{r}$ is strictly inside throughput region we infer $r = \sum_{I^k \in S} \alpha_k I^k$ where $\sum_{k=1}^{|S|} \alpha_k < 1$. Then,

$$Q(t).r = \sum_{I^k \in S} \alpha_k Q(t).I^k \leq \sum_{I^k \in S} \alpha_k Q(t).I^*(t) < Q(t).I^*(t)$$

A useful candidate for $\boldsymbol{r}$ is what we have derived in relation (8). By applying (8) in Lemma 1, we get the following result:

$$\begin{aligned} &\boldsymbol{Q}.(\boldsymbol{\lambda} + \frac{1-\rho}{\chi'(G)} \mathbf{1}_L) < \boldsymbol{Q}.\boldsymbol{I}^* \\ &\boldsymbol{Q}.(\boldsymbol{\lambda} - \boldsymbol{I}^*) < -(\frac{1-\rho}{\chi'(G)}) \mathbf{1}_L.\boldsymbol{Q} \end{aligned} \tag{10}$$

Now we are ready to provide the main result of this section.

Theorem 2 *Under 1-hop interference mode, if* $\boldsymbol{\lambda}$ *is strictly inside the throughput region, then the average delay of the network is upper bounded by* $\frac{\chi'(G)(\sum_{i=1}^{L}[Var(A_i)+\lambda_i-\lambda_i^2])}{2(1-\rho)\sum_{i=1}^{L}\lambda_i}$, *where* ρ *is the loading factor and* $\chi'(G)$ *is the edge-chromatic number of the network graph.*

Proof To prove this theorem, we apply the following Lyapunov function in Theorem 1.

$$V(\boldsymbol{Q}) \triangleq \boldsymbol{Q}.\boldsymbol{Q} = \sum_{i=1}^{L} Q_i^2 \tag{11}$$

$$\begin{aligned} &\mathbb{E}[V(\boldsymbol{Q}(t+1)) - V(\boldsymbol{Q}(t))|\boldsymbol{Q}(t)] = \\ &\mathbb{E}[(\boldsymbol{Q}(t+1) - \boldsymbol{Q}(t)).(\boldsymbol{Q}(t+1) + \boldsymbol{Q}(t))|\boldsymbol{Q}(t)] \\ &= \mathbb{E}[(\boldsymbol{A}(t) - \boldsymbol{I}(t)).(2\boldsymbol{Q}(t) + \boldsymbol{A}(t) - \boldsymbol{I}(t))|\boldsymbol{Q}(t)] \\ &= 2\mathbb{E}[\boldsymbol{Q}(t).(\boldsymbol{A}(t) - \boldsymbol{I}(t))|\boldsymbol{Q}(t)] + \\ &\quad \mathbb{E}[(\boldsymbol{A}(t) - \boldsymbol{I}(t)).(\boldsymbol{A}(t) - \boldsymbol{I}(t))|\boldsymbol{Q}(t)] \\ &= 2Q(t).\mathbb{E}[(\boldsymbol{A}(t) - \boldsymbol{I}(t))|\boldsymbol{Q}(t)] + \\ &\quad \mathbb{E}[(\boldsymbol{A}(t) - \boldsymbol{I}(t)).(\boldsymbol{A}(t) - \boldsymbol{I}(t))|\boldsymbol{Q}(t)] \\ &= 2Q(t).\{\mathbb{E}[\boldsymbol{A}(t)|\boldsymbol{Q}(t)] - \mathbb{E}[\boldsymbol{I}(t)|\boldsymbol{Q}(t)]\} + \\ &\quad \mathbb{E}[(\boldsymbol{A}(t) - \boldsymbol{I}(t)).(\boldsymbol{A}(t) - \boldsymbol{I}(t))|\boldsymbol{Q}(t)] \end{aligned} \tag{12}$$

We know that Q(t) depends on arrivals and departures till time slot t (before time slot t). Therefore, it is independent of arrivals at time slot t. Thus, $\mathbb{E}[\boldsymbol{A}(t)|\boldsymbol{Q}(t)] = \mathbb{E}[\boldsymbol{A}(t)] = \boldsymbol{\lambda}$. Since we apply Max-Weight schedule at each time slot for the given $Q(t)$, and if there are more than one optimal $I(t)$, for all of them the value of $\boldsymbol{Q}(t).\boldsymbol{I}(t)$ is constant, then Equation (12) can be rewritten as follows.

$$\begin{aligned} &\mathbb{E}[V(\boldsymbol{Q}(t+1)) - V(\boldsymbol{Q}(t))|\boldsymbol{Q}(t))] \\ &= 2[\boldsymbol{Q}(t).(\boldsymbol{\lambda} - \boldsymbol{I}(t))] + \mathbb{E}[g(t)|\boldsymbol{Q}(t)] \end{aligned} \tag{13}$$

where $g(t) \triangleq (\boldsymbol{A}(t) - \boldsymbol{I}(t)).(\boldsymbol{A}(t) - \boldsymbol{I}(t))$. Using relation (10) in the first term of (13) we infer,

$$\begin{aligned} &\mathbb{E}[V(\boldsymbol{Q}(t+1)) - V(\boldsymbol{Q}(t))|\boldsymbol{Q}(t))] \\ &\leq -2(\frac{1-\rho}{\chi'(G)}) \mathbf{1}_L.\boldsymbol{Q}(t) + \mathbb{E}[g(t)|\boldsymbol{Q}(t)] \end{aligned} \tag{14}$$

Now, by plugging the above inequality into Theorem 1 we have,

$$\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{i=1}^{L} \mathbb{E}[Q_i(\tau)] \leq \frac{\chi'(G)}{2(1-\rho)} \overline{g} \tag{15}$$

where $\overline{g} \triangleq \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[g(\tau)]$.

Note that if the arrival process have bounded second moment, then $\overline{g} < \infty$ and the system is strongly stable. Therefore, for each link l, the long term average of service rate equals its arrival rate, i.e., $\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} I_i(\tau) = \lambda_i$. Also, $I_i^2(t) = I_i(t)$ because $I_i(t) \in \{0, 1\}$. Then,

$$\begin{aligned} \overline{g} &= \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[\boldsymbol{A}(\tau).\boldsymbol{A}(\tau) + \boldsymbol{I}(\tau).\boldsymbol{I}(\tau) - 2\boldsymbol{A}(\tau).\boldsymbol{I}(\tau)] \\ &= \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[\sum_{i=1}^{L} A_i^2(\tau) + \sum_{i=1}^{L} I_i^2(\tau)] - 2\sum_{i=1}^{L} \lambda_i^2 \\ &= \sum_{i=1}^{L} Var(A_i) + \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E} \sum_{i=1}^{L} I_i(\tau) - \sum_{i=1}^{L} \lambda_i^2 \\ &= \sum_{i=1}^{L} [Var(A_i) + \lambda_i - \lambda_i^2] \end{aligned} \tag{16}$$

Following the same spirit of works in [19,20], we know that the queue backlogs evolve according to an ergodic Markov chain with a countably infinite state space. Thus, the left hand side of (15) can be replaced by time average total queue backlog in the network which is denoted by $\overline{Q}$. Then after plugging (16) in (15) we have,

$$\sum_{i=1}^{L} \overline{Q_i} \leq \frac{\chi'(G)(\sum_{i=1}^{L}[Var(A_i) + \lambda_i - \lambda_i^2])}{2(1-\rho)} \tag{17}$$

The average expected delay in the network, $\overline{W}$, is derived by Little's Law as follows,

$$\overline{W} \leq \frac{\chi'(G)(\sum_{i=1}^{L}[Var(A_i) + \lambda_i - \lambda_i^2])}{2(1-\rho)\sum_{i=1}^{L}\lambda_i} \tag{18}$$

4 Simulation results

We have simulated the Max-Weight scheduling algorithm, using the cycle and complete graphs as our network topologies. In complete graph every pair of distinct nodes is connected by an edge. We set up three different topologies. Let *Complete* 8 denotes a complete graph with 8 nodes, *Cycle* 8 denotes a cycle with 8 nodes and *Cycle* 16 denotes a cycle with 16 nodes. In Fig. 1, the average delay of two cycle topologies in addition to our calculated upper bound are depicted versus the increasing arrival rate. Note that the derived upper bound for *Cycle* 8 and *Cycle* 16 are almost the same. In order to understand the accuracy of obtained delay and its upper bound better, a lower bound is computed using the results of a recent paper [20] in which authors have considered a fictitious scheduling algorithm to derive this bound. In the same paper, an upper bound for delay has been derived for a generalized version of Max-Weight schedule, termed as GMWM. The bound is not applicable for Max-Wight schedule and also its calculation requires to solve a convex optimization problem using complicated methods such as Lagrange multipliers and iterative subgradients. But our proposed method uses the graph edge-chromatic number to calculate the upper bound which simplifies the computation. (refer to Sect. 6 for details of discussion).

It can be observed that the delay of the two cycle topologies are almost the same and are independent of the number of nodes or links. Note that the edge-chromatic number of these two cycles are the same and are equal to 2 (We can label all the edges by 2 colors such that adjacent edges get different colors).

Fig. 2 shows the increase in delay of *Complete* 8 topology as the load is increased. In Fig. 3, the average delay of *Cycle* 16 is compared to the average delay of *Complete* 8. It is obvious that the delay of the complete graph is larger than the delay of cycle topology, due to its larger edge-chromatic number which is 7 here. Note that the increment in delay is not necessarily proportional to $\chi'(G)$. In the special case of a symmetric system with identical input rates, Max-Weight scheduling minimizes the average delay of the network [16]. This property has been shown by simulation in [20], where Max-Weight policy performs close to the derived lower bound of delay. It is necessary to mention that neither measuring the capacity region of an arbitrary network, nor running Max-Weight scheduling algorithm for it is a straightforward task. This is why, we use simple symmetric graphs in our simulation.

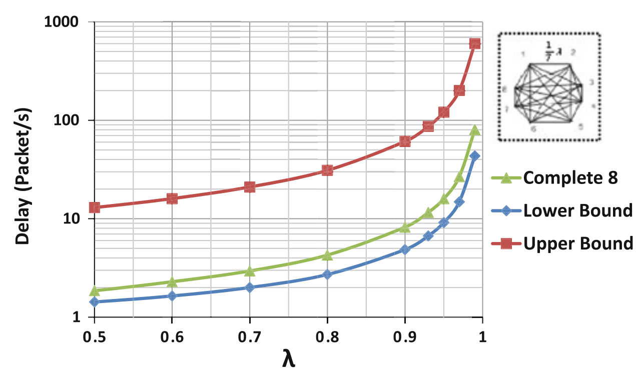

Fig. 2 Delay of complete topology

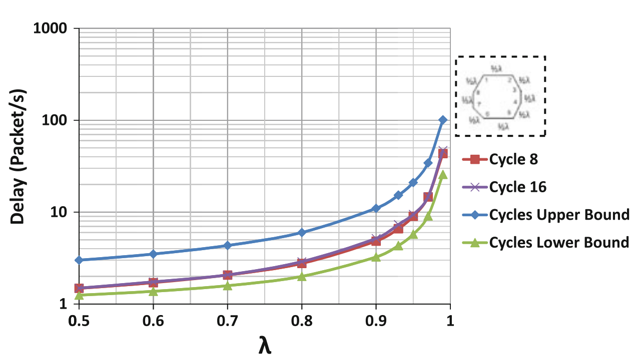

Fig. 1 Delay of cycle topology

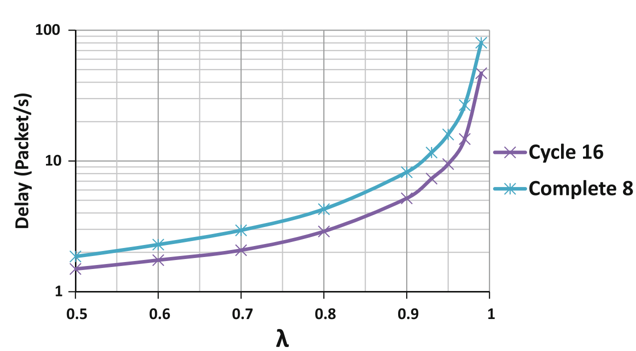

Fig. 3 Delay comparison of cycle and complete topologies

5 Extension to general interference model

Theorem 2 in Sect. 3 is applied to the 1-hop interference model. In this section we aim to extend the result of Theorem 2 to the case of general interference model. To consider general interference model, we use the notion of conflict graph, G' (refer to Section 2 for definition). Note that there is a one to one relation between feasible schedules in graph G and independent sets in graph G'.

Now, assume that the nodes of G' has been labeled by $\chi(G')$ colors, where $\chi(G')$ is the chromatic number of G' (refer to Sect. 2 for definition). Each color corresponds to a feasible schedule in the graph. Therefore, we have a set of feasible schedules Γ which consists of $\chi(G')$ schedules. The union of active links in Γ equals $\mathcal{L}$ and any link belongs to only one schedule in the set. Note that $\Gamma \subset \mathcal{S}$, where $\mathcal{S}$ is the set of all feasible schedules. Let $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_{\chi(G')}\}$, where γ_i is a schedule corresponds to color i, $1 \le i \le \chi(G')$. Note that $\frac{1}{\chi(G')}\sum_{i=1}^{\chi(G')} \gamma_i \in \Lambda$ due to the convex property of Λ. Now, similar to the approach of Sect. 3 and by replacing $\chi'(G)$ by $\chi(G')$, we can derive the following theorem.

Theorem 3 *Under general interference model, if* $\boldsymbol{\lambda}$ *is strictly inside the throughput region, then the average delay of the network is upper bounded by* $\frac{\chi(G')(\sum_{i=1}^{L}[Var(A_i)+\lambda_i-\lambda_i^2]}{2(1-\rho)\sum_{i=1}^{L}\lambda_i}$.

Proof The proof parallels the development in Theorem 1, in which $\chi(G')$ is used instead of $\chi'(G)$ and then is omitted due to similarity.

6 Discussion

We have shown that the average delay of packets in wireless network with single hop traffic flows under 1-hop interference model is upper bounded by edge-chromatic number. The significance of this result becomes more prominent if it is pointed out that there are precisely two classes of graphs, those with $\chi'(G) = \Delta$ and those with $\chi'(G) = \Delta + 1$ [24] where Δ is the largest vertex degree of the graph. In other words, we need at least Δ and at most $\Delta + 1$ colors to color (edge coloring) the graph.

Although calculating the precise value of $\chi'(G)$ for an arbitrary graph is an NP hard problem itself, but for our purpose we might use $\Delta + 1$ to simply compute the delay upper bound without any complex calculations.

We notice that Δ can be any integer value in the interval $[1, N-1]$, where N is the number of nodes in the graph. In special case of large and sparse graphs where nodes are distributed uniformly, Δ is typically small compared to N. Then we can infer that delay bound is order optimal as it is developed in [18]. Indeed, our new scheme elegantly generalizes the result of [18] in the sense that we have derived an upper bound which explicitly reflects the impact of network topology on delay.

7 Conclusions

In this paper, we have studied the effect of network topology on delay of throughput optimal Max-Weight link scheduling algorithm in wireless networks with single hop traffic flows. Based on the interference model we have used, two different bounds are obtained. We have first derived an upper bound for the delay under 1-hop interference model in terms of edge-chromatic number of network graph and loading factor. The result is of interest due to an interesting property of graphs that the Edge-chromatic number is either Δ or $\Delta + 1$, where Δ is the largest vertex degree of the graph and can be obtained easily from network topology. Then, another upper bound for delay under general interference model has been established in terms of chromatic number of network conflict graph and loading factor. We believe that the results of this paper reveals the relation between average delay and network topology in the order sense and add to the understanding of the impact of topology on delay.

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